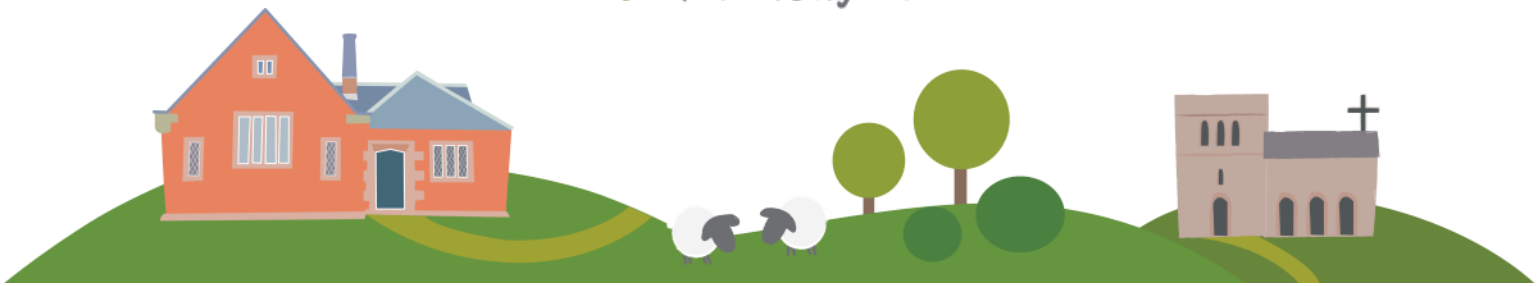


Raughton Head C of E Primary School

Calculation Policy

September 2022

"A high-quality mathematics education provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject." Department for Education, 2013

The following pages show Raughton Head C of E School's calculation policy (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) approach across mathematics in school helps children develop a mastery approach across all the operations in an efficient and reliable way. This policy shows how these methods develop children's confidence in their understanding of both written and mental methods.

Mastery Approach

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. All children should have access to the same curriculum content where possible, and rather than being extended with new learning, they should deepen their conceptual understanding by applying their knowledge using challenging problems. Children should demonstrate their understanding of the calculation strategies through the use of concrete materials and pictorial representations.

Mathematical Language

The National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning. The non-statutory guidance highlights the requirements for children to extend their language around certain concepts. It is essential that teaching using the strategies outlined in this policy are accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential and must be consistent across the school. **The school agreed list of terminology is located at Appendix A to this document.**

How to use the policy

This calculation policy is a guide for all staff and has been adapted to meet the requirements of our school. It is purposely set out as a progression of mathematical skills and not into year group phases to encourage a flexible approach to teaching and learning that can run across our mixed-age classes. Despite this, the expectations for each key stage are laid out below for teachers to refer to, to ensure expected and appropriate coverage. It is expected that teachers will use their professional judgement as to when consolidation of existing skills is required or if to move onto the next concept. The focus must always remain on breadth and depth rather than accelerating through concepts. Children should not be extended with new learning before they are ready, they should deepen their conceptual understanding by applying learning to problems. New or alternative methods should be shown to children alongside methods that they have previously learnt.

Teachers have access to Hamilton Trust planning and can adapt this accordingly to meet the needs of their children. Teachers can use any teaching resource that they wish to use and this policy does not recommend one set of resources over another, rather that, a variety of resources are used.

For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations.

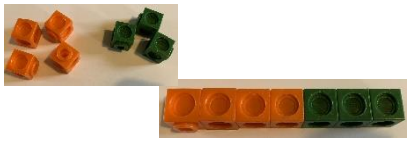
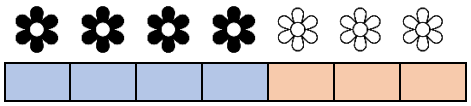
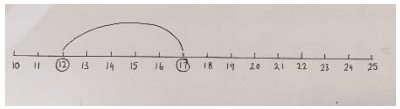
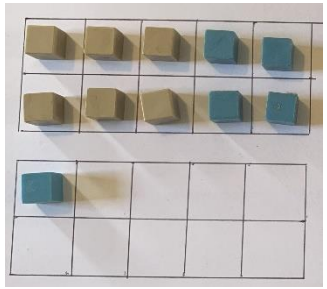
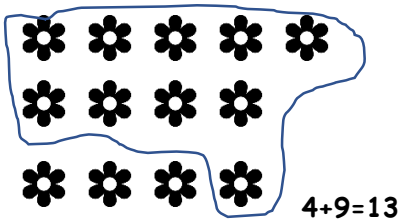
EYFS		
Children become familiar with number through play, games and songs. Regular, short lessons allow children to develop a positive relationship with number and calculation. Activities are hands-on and concrete resources are available for children to investigate.		
Key language: count up, count down, count on, count back; count in ones, twos, fives, tens, same as, more, less, few, pattern, pair, digit, ones, tens, as many as, more, larger, bigger, greater, fewer, smaller, less, most, biggest, largest, greatest, one more, ten more, one less, ten less, compare, order, size; first, second, third, last, last one but, before, after, next, between, Add, more, and, make, sum, total, altogether, double; one more, two more, ten more, take away, one less, sharing, doubling, halving,		
Number, addition and subtraction: Children will have a deep understanding of number to 10, including the composition of each number. They can subitise (recognise quantities without counting) up to 5 and can automatically recall number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. Children will count beyond 20. They will compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as another quantity.	Multiplication and division: Children will explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	

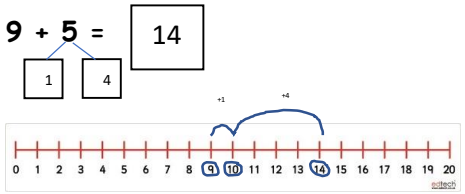
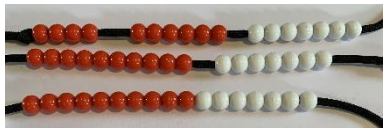
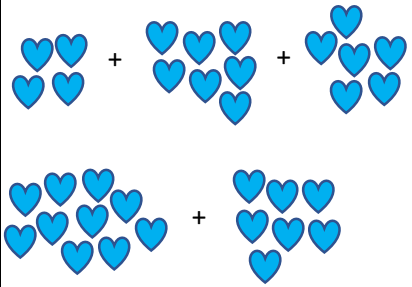
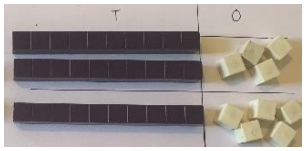
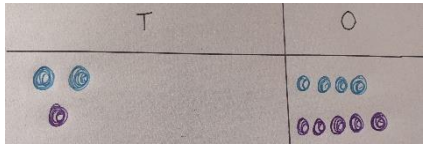
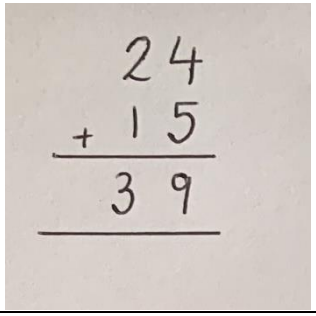
Key Stage 1		
Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.		
Key language: whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table		
Addition and subtraction: Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations. A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 – 3 and 15 – 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods. In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.	Multiplication and division: Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.	Fractions: In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole. In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator

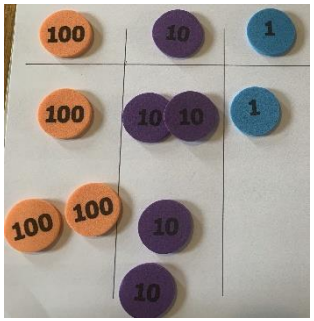
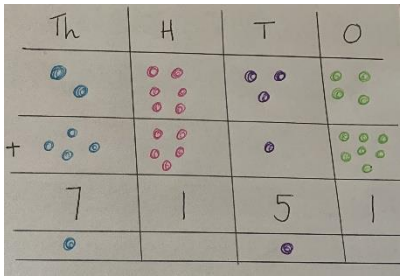
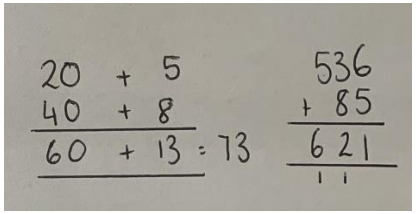
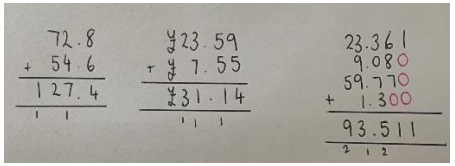
<table>
<tr><th colspan="3">Lower Key Stage 2</th></tr>
<tr><td colspan="3">In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.</td></tr>
<tr><td colspan="3">Key language: partition, place value, tens, hundreds, thousands, column method, whole, part, equal groups, sharing, grouping, bar model</td></tr>
<tr><td>Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.</td><td>Multiplication and division: Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.</td><td>Fractions: Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside. in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1. Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.</td></tr>
</table>

Upper Key Stage 2		
In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.		
Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number		
Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.	Multiplication and division: Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6.	Fractions: Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole	Use cubes to add two numbers together as a group or in a bar. 	 Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 7 = 4 + 3 Put larger number in your head and count on smaller number to find your answer using your fingers or in your head.
Starting at the bigger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer. 	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer
Regrouping to make 10	Start with the bigger number and use the smaller number to make 10. Add on the remainder. 6+5=11 	Use pictures or a number line. Regroup or partition the smaller number to make 10.  4+9=13	7 + 4 = 11 If I am at seven, how many more do I need to make 10? (3) How many more do I need to add on? (1)

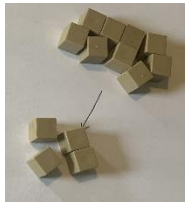
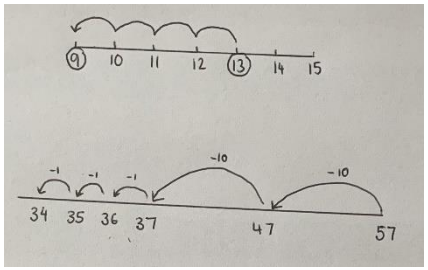
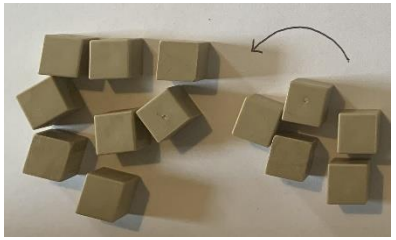
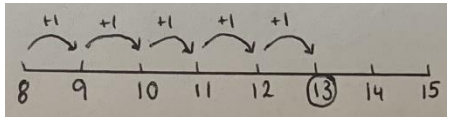
			
Adding three single digits	4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) and then add on the third digit.	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	4 + 7 + 6 = 10 + 7 = 17 If possible, add the two numbers that make 10 and then add on the final number. 4 + 7 + 4 = 11 + 4 = 15 If not possible to make 10, use above strategies. Put largest number in your head and count on a smaller number. Add on remainder.
Column method – no carrying/regrouping **(Year 3 onwards)**	24 + 15 = 39 Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. 	After practically using the Base 10 blocks and place value counters, children can draw the counters to help them to solve addition. 	

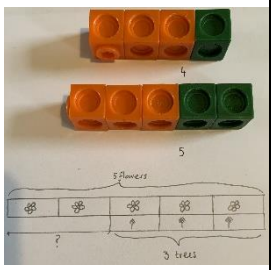
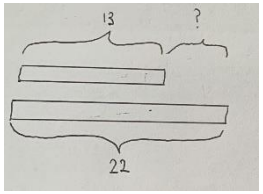
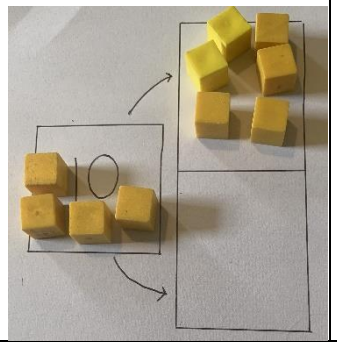
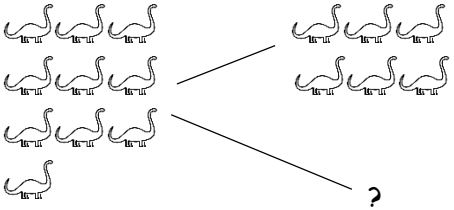
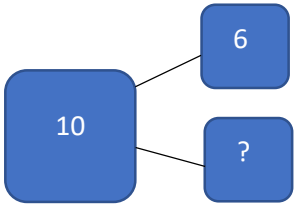
			
Column method – with carrying/regrouping	Make both numbers on a place value grid.  **Add up the units and exchange 10 ones for one 10.**  Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.  	Start by portioning the numbers before moving on to clearly show the exchange **below** the addition.  As the children move on, combine the numbers and complete the addition without portioning. Introduce decimals with the same number of decimal places and different – money can be used here. 

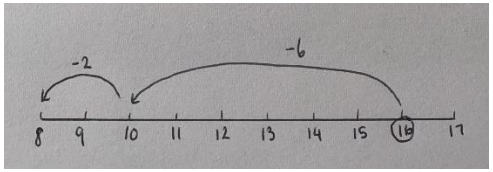
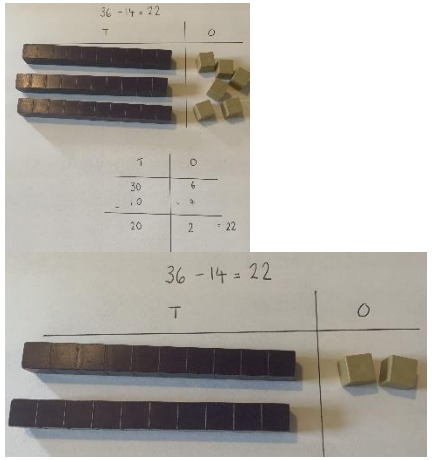
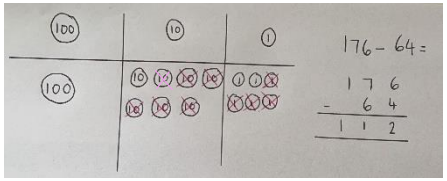
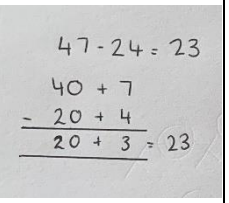
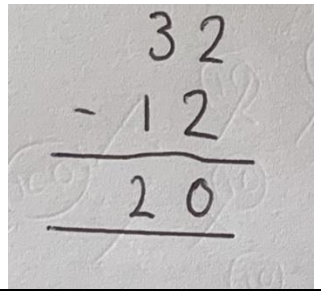
	This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move onto decimals, money and decimal place value counters can be used to support learning.		

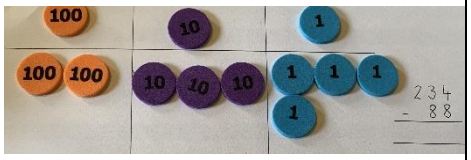
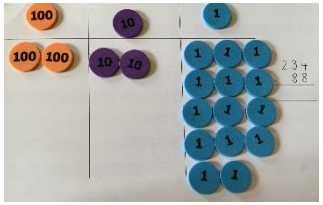
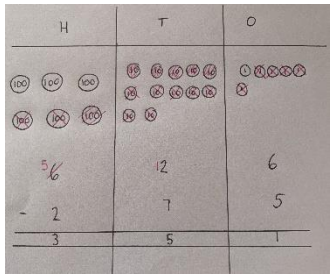
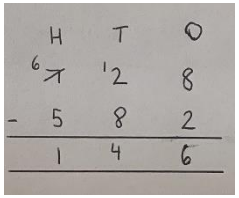
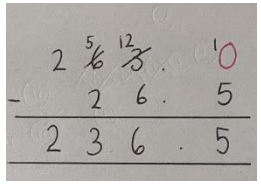
Subtraction

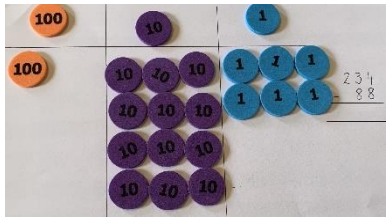
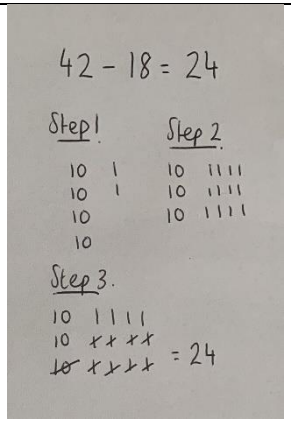
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc. to show how objects can be taken away. 8 – 2 = 6 	Cross out drawn objects to show what has been taken away.  8 – 2 = 6	8 – 2 = 6 18 – 2 = 16 Put the larger number in your head/on your fingers. Count back using the smaller number. Remember not to begin counting back with original number.
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count back in ones. What are you left with?	Count back on a number line. 13 – 4 = 9	13 – 4 = 9 Put 13 in your head, count back 4. What number are you at? Use your fingers to help if needed.

	13 – 4 = 9  Use counters and move them away from the group as you take them away counting backwards as you go. 	Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2-digit numbers. 57 – 23 = 34	
Counting on	Start with the number you are taking away (8). Collect this amount of counters. Add counters in ones until you reach your starting number (13). What number did you get? 13 – 8 = 5 	On a number line, start at the number you are taking away (8). Count on in ones showing the jumps on the number line. How many ones have you added to get back to the starting number? 	13 – 8 = ? Put 8 in your head and count on to 13. How many have you counted on? 5

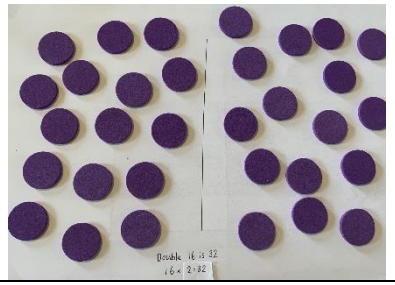
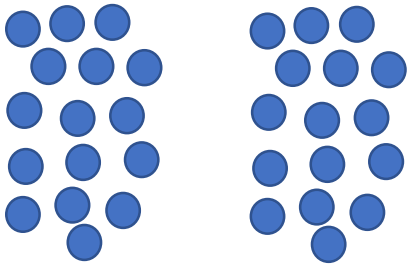
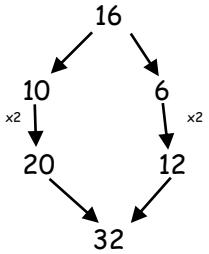
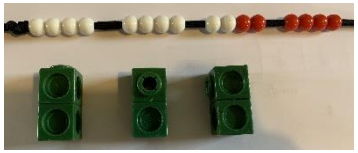
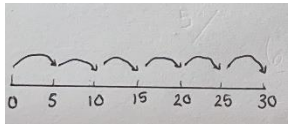
Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference. 	Use a number line. Count on (as above) to find the difference. Draw bars to find the difference between 2 numbers. *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.* 	Hannah has 23 sandwiches. Helen has 15 sandwiches. Find the **difference** between the number of sandwiches. 23 – 15 = 8
Part whole model	Link to addition to help explain the connection between addition and subtraction (inverse) If 10 is the whole and 6 is one of the parts, what is the other part? 10 – 6 = 4 	Use a pictorial representation of objects to show the 'part whole model' 	Move to using numbers within the part whole model 

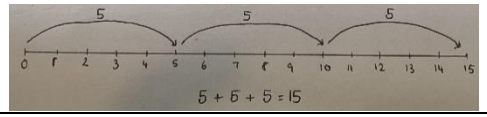
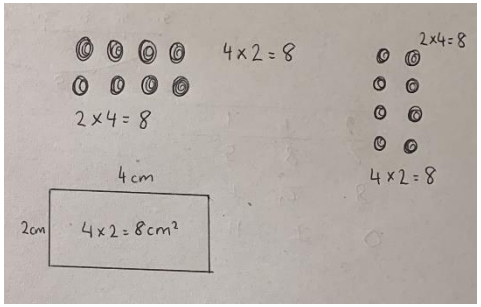
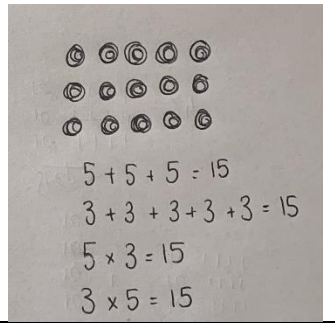
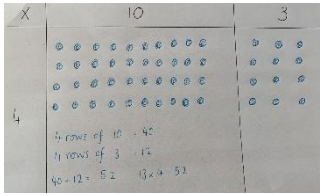
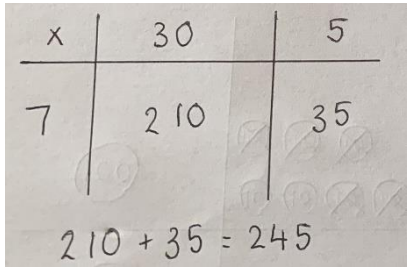
Make 10	16 – 8 = 8  Make 16 on the ten frame. Take away the 6 first to make 10 and then takeaway 2 more so you have taken away 8. You are left with the answer of 8.	Start at 16. Take 6 away to reach 10. Then take away the remaining 2 so you have taken away 8 altogether. You have reached your answer (8). 	16 – 8 = 8 How many do we take off to reach the previous 10? 6 How many more do we have left to take off? 2 Answer = 6 + 2 = 8
Column method without regrouping/exchanging **(Year 3 onwards)**	Use Base 10 to make the bigger number then take away the smaller number.  Show how you partition numbers to subtract. Make the larger number first.	Draw the Base 10 or place value counters alongside the written calculation to help show working. 	47 – 24 = 23 This will lead to a clear written column subtraction.  

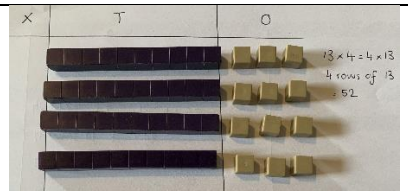
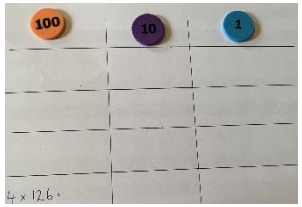
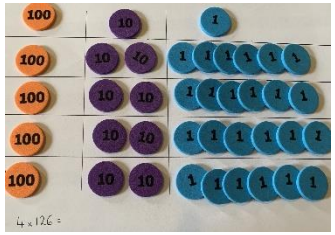
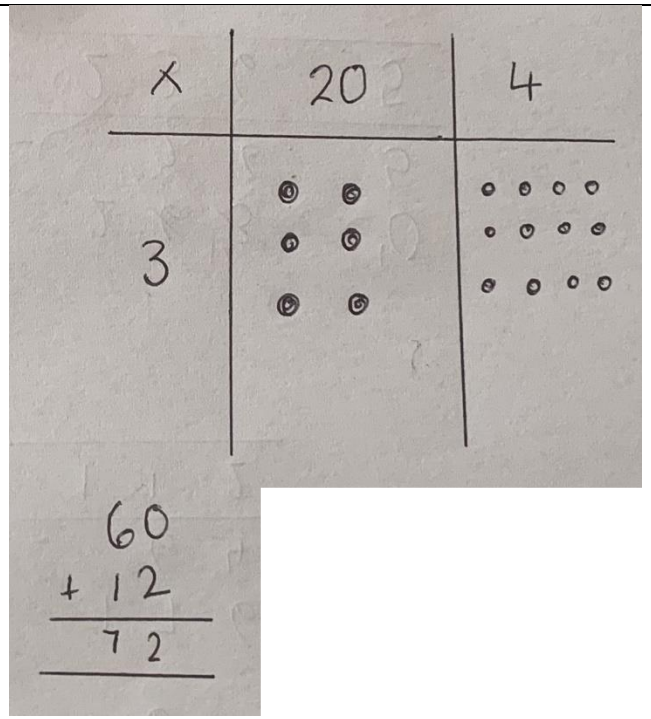
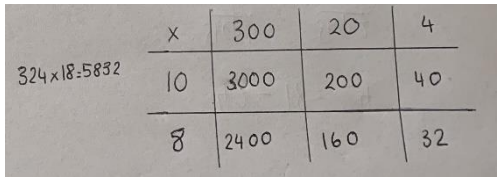
Column method with regrouping/exchanging	234 – 88 = Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtraction with 2 exchanges. Make the larger number with the place value counters.  Start with the ones. Can I take away 8 from 4 easily? No. I need to exchange one of my tens for ten ones.  Now I can subtract my ones. 	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you have made.  When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here demonstrates the child understands the method and knows when to exchange/regroup.	Children can start their formal written method by portioning the number into clear place value columns. Moving forward, the children can use a more compact method.  This will lead to an understanding of subtracting any number including decimals. 

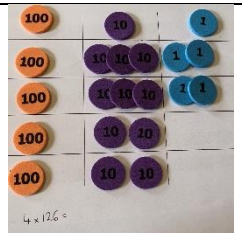
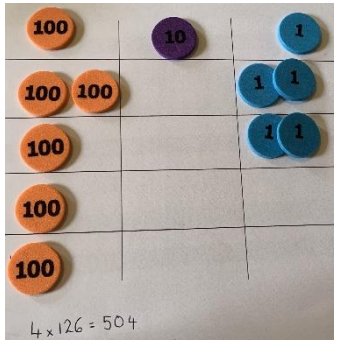
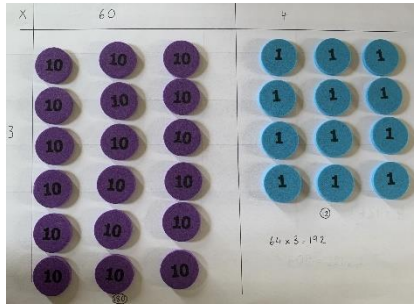
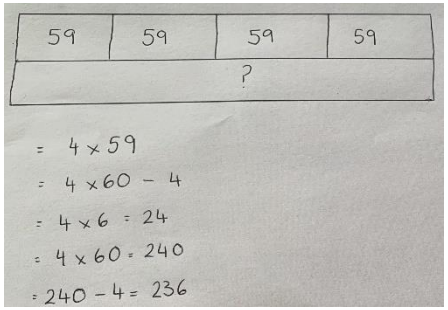
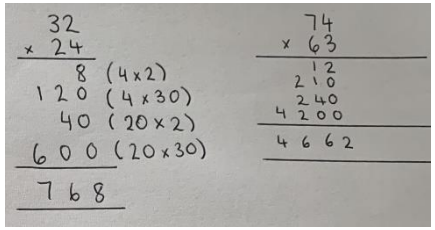
	Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction.  Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging show where we write our new amount (to the top left of the next digit).		

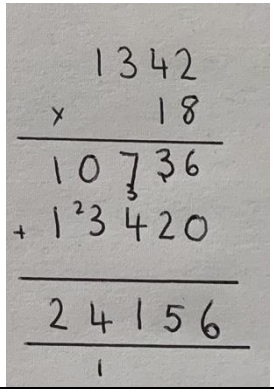
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical resources such as counters or cubes to show how to double a number. Double 16 – get 16 counters out and then 16 more. Add together. 	Draw pictures to show how to double a number. Double 16 is 32 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	Count in multiples supported by concrete objects in equal groups. 	Use a number line or picture to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25
Repeated addition	Use different objects to add equal groups. 	*There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there in total?* 2 add 2 add 2 equals 6 	Write addition sentences to describe objects and pictures.  2 + 2 + 2 = 6

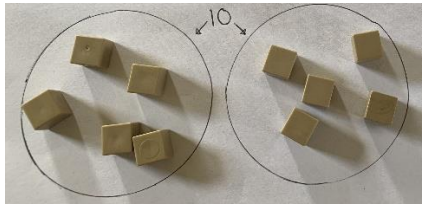
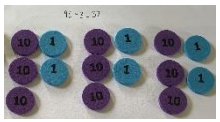
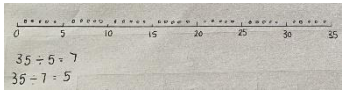
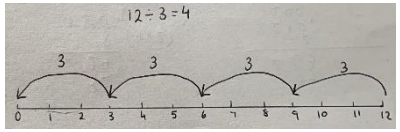
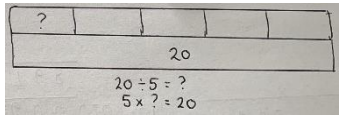
			
Arrays – showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences. 	Draw arrays in different rotations to find different **commutative** multiplication sentences. Link arrays to areas of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition. 
Grid method	Show the link with arrays to first introduce the grid method.  Move on to using Base 10 to move towards a more compact method.	Children can represent the numbers with place value counters in a way that they understand. They can draw counters, use colours to show different amounts or use circles to show their thinking. See below.	Start with multiplying by 1-digit. Show the addition alongside the grid. 

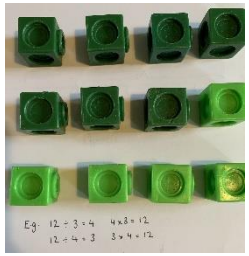
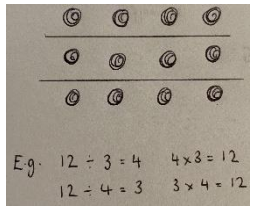
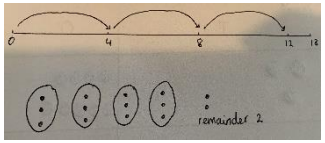
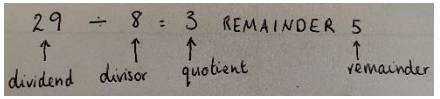
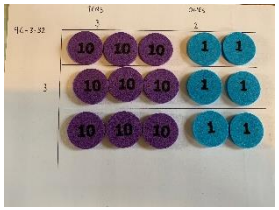
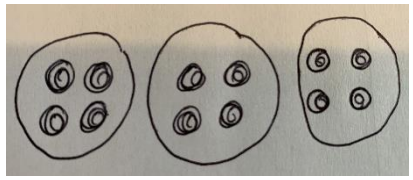
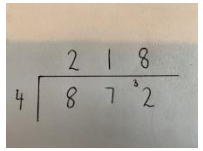
	 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows. Fill each row with 126.   Add up each column, starting with the ones. Make any exchanges needed.		Moving forward, multiply by a 2-digit number (and then higher), showing the different rows within the grid method. 

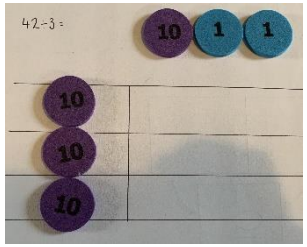
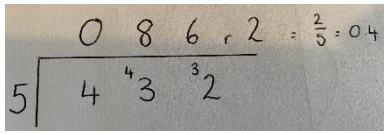
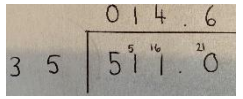
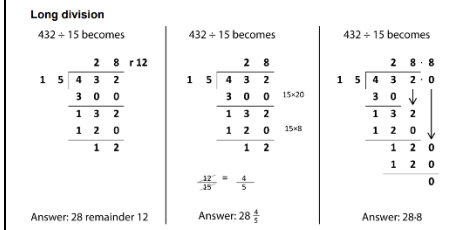
	 Then you have your answer. 		
Column multiplication **(Year 3 onwards)**	Children can continue to be supported by place value counters at this stage of multiplication. 	Bar modelling and number lines can support learners when solving problems involving multiplication alongside written methods. 	Where necessary, start with long multiplication. Remind the children about lining up their numbers clearly in columns. If it helps, children can write out the question they are solving next to their answer. 

	It is important at this stage that they always multiply the ones first and note the answer down, followed by the tens.		Then move on to short multiplication (teachers may decide to move straight to this method depending on children's capability). Carried numbers are written directly next to answer as seen in the example calculation. 

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	I have 10 cubes. Can you share them equally in to 2 groups? 	Children use pictures or shapes to share quantities. 8 ÷ 2 = 4 	Share 9 buns between 3 people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cube, counters, objects or place value counters to aid understanding.   	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. Use multiplication skills to do this. 	28 ÷ 7 – 4 Divide 28 into 7 groups. How many are in each group?

Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. 	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7
Division with a remainder	14 ÷ 3 = Divide objects between groups and see how many are left over. 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount. Remainder will be shown clearly.	Complete written divisions and show the remainder using 'r'. 
Short division/Long division **(Year 3 onwards)**	 Use place value counters to divide using the bus stop method alongside. Start with the biggest place value. We are sharing 40 into three groups. We can put 1	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. 	Begin with divisions that divide equally with no remainder. 

	ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how many in one group. The answer is 14.	Encourage them to move towards counting in multiples to divide more efficiently.	Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  <u>Long Division</u> 

Appendix A – Correct Mathematical Language

High expectations of the mathematical language used are essential. Consistency across the school is key.

Correct Terminology	Incorrect Terminology
Ones	Units
Is equal to (is the same as)	Equals
Zero	'Oh' (the letter o)
Exchange Exchanging Regrouping (carrying)	Borrowing Stealing
Calculation Equation	Sum Number sentence
Known Unknown	
Whole Part	

See examples as to where to place digits as to ensure consistency across calculations throughout the school. E.g. When regrouping forward in addition calculations, digits will be placed at the bottom of the calculation (rather than in the calculation/above the calculation) to keep carried digits separate from the other digits.